EFFECT OF VORTEX SHEDDING ON THE STRUCTURAL INTEGRITY OF KURZ INSERTION MASS FLOW ELEMENTS

I. INTRODUCTION

Equations are developed to calculate the vibration and structural characteristics of the Kurz Series 450 and K-BAR Insertion Mass flow Elements. These include the natural frequency of the sensor support tube, the vortex shedding frequency, and stress caused by vortex shedding.

Vortex shedding occurs over a Reynolds Number range of approximately 50 to 5000 for circular cylinders. Vortex generation above a Reynolds Number of 5000 is not stable and may not cause resonance problems. In fact, Murdock (ASME Journal October 1959) stated that all attempts to obtain resonance during experimental tests failed. One reason is that shedding over all portions of a long cylindrical bar is not synchronized.

Table 1 gives the lower and upper free stream velocity for three sizes of Insertion Mass flow Elements manufactured by Kurz Instruments to cause vortex shedding.

PRODUCT	SUPPORT DIAMETER	VELOCITY (FPM) UNDER STANDARD CONDITIONS	
		Re = 50	Re = 5000
Series 450-08	1/2"	11.75	1175
Series 450-16	1"	5.87	587
Series K-BAR24	1 1/2"	3.92	392

Table 1

The table shows that vortex shedding occurs at very low velocities, such that the velocity pressure is also very small. This means that the stress produced, even near resonance is also very small compared to the allowable stress of 30,000 PSI for stainless steel, at least for low pressure and temperature situations normally encountered n thermal mass flow meter applications. If liquid or extremely high pressures were used at very high velocities, one would have reason for concern. Since these situations generally have Reynolds Numbers much larger than 5000, our conclusion is that the effect is greatly exaggerated as a failure mechanism, at least for our type of instrument.

II ASSUMPTIONS

1) The force caused by vortex shedding is equal to drag caused by the gas velocity pressure, where the drag coefficient C_D = 1.0 (Data shows this is very conservative).

2) The Damping Factor is between .01 and .05 per the Engineering Handbook.

3) The sensor support window is considered to be a solid tube, although it is open and gas flow through it (this is also a conservative assumption).

4) $E = 29 \times 10^6$ PSI.

5) $\sigma_{ALLOW} = 30{,}000$ PSI.

6) Material is 316SS or equal, density = 488 lb/Ft^3.

7) Sensor support is cantilever mounted, with no end support.

8) Vortex shedding occurs above a Reynolds Number of 5000, although this is unsupported in the literature and highly conservative.

III GENERAL OBSERVATIONS

1) Because of typical support size, length, and typical velocities in ducts, the vortex frequency is normally considerably higher than the natural frequency of the support and the force magnification factor is very small. The gas velocity pressure at resonance is so low that forces are negligible (the force is measured inches of water because of the low velocity).

2) We have not encountered a resonance condition caused by vortex shedding at any of our thousands of installations, nor have we had a failure due to vortex shedding.

3) We essentially followed basic assumptions of the ASME Power Test Code Method for thermowells by J.W. Murdock (October 1959), however, our problem is quite different in that the velocity pressure is very small, as are the temperature and gas pressures.

4) In most cases the vortex shedding frequency is much higher than the natural frequency of the sensor support, as is the case for automobile antennas. I think we can safely say that automobile antennas do not exhibit a measurable failure rate due to vortex shedding!

IV EQUATIONS AND TERMS

A) Equations

1) $$f_N = \frac{22{,}418\sqrt{D^2 + d^2}}{L^2}$$

2) $$p_a = .0748\left(\frac{MW}{29}\right) x \left(\frac{P_a}{P_S}\right)\left(\frac{T_S}{T_a}\right)$$

3) $$f_w = \frac{2.64\ m}{P_a\ A\ D}$$

4) $$F_M = \frac{1}{\left(1 - \left(\frac{f_W}{f_N}\right)^2\right)^2 + \delta^2}$$

5) $$\delta_{ST} = \frac{1.576\ x\ 10^4\ (L - .5L_1)\ L_1\ D^4\ \rho_a\ f_W^2}{(D^4 - d^4)}$$

6) $\delta_{DYN} = \delta_{ST} \pm F_M \, \delta_{ST}$

7) $\delta_g = \dfrac{1.13 \; L^2 \; D}{D^2 + d^2}$

8) $\delta_{MAX} = \delta_g + \delta_{DYN}$

B) Terms

D = Sensor Support Diameter (inches)

d = Inside Diameter of Sensor Support (inches)

L = Unsupported Length of Sensor Support (inches)

A = Flow Area of Duct or Stack (FT^2)

m = Mass of Gas (lb$m$/sec)

ρ_a = Actual Density of Gas (lb$m$/FT^3)

P_a = Absolute Pressure of Gas (PSIA)

P_s = Absolute Pressure of Gas at Standard Conditions (14.7 PSIA)

MW = Gas Molecular Weight

T_S = Standard Temperature, 537 °R

T_a = Actual Temperature of Gas °R

δ = Damping Factor, (.01 - .05)

f_W = Vortex Shedding Wake Frequency (Hz)

f_N = Natural Frequency of Sensor Support (Hz)

F_M = Force Magnification Factor

L_1 = Length of Sensor Exposed to Flow (Inches), Includes Shield

δ_{ST} = Maximum Stress at Sensor Support (PSI) Caused by Gas Flow Force

δ_{DYN} = Stress at Sensor Support (PSI) Caused by Vortex Shedding

δ_g = Stress at Sensor Support Caused by Weight of Support

δ_{MAX} = Sum of Stresses, δ_{DYN}, δ_g

Typical Series 450 Geometry

V. EXAMPLES

Example A: Moderate Flare Gas Application Velocity

Process Pipe has 23.25" Diameter. $A = 2.948\ Ft^2$

D = 1", Series 450-16 Insertion Mass Flow Element

d = 0.87"

L = 19.54"

L_1 = 4.2" (Centerline of sensor placed at equal area location of pipe)

m = 23.9 lb$m$/sec

P_a = 7.8 PSI = 22.5 PSIA

P_S = 14.7 PSIA

MW = 54.63

T_a = 198 °F = 658 °R

T_S = 537 °R

δ = .01

Results:

1) Natural Frequency

$$f_N = \frac{22{,}418 \sqrt{D^2 + d^2}}{L^2} = 77.83 \ Hz$$

2) Gas Density

$$\rho_a = .0748 \left(\frac{MW}{29}\right) x \left(\frac{P_a}{P_S}\right) \left(\frac{T_s}{T_a}\right) = .1760 lbm/FT^3$$

3) Vortex Shedding Frequency

$$f_W = \frac{2.64 \ m}{\rho \ A \ D} = 121.61 \ Hz$$

4) Force Magnification Factor

$$F_M = \frac{1}{\sqrt{\left(1 - \left(\frac{f_W}{f_N}\right)^2\right)^2}} + \delta^2 = .69374$$

5) Maximum Stress at Sensor Support (PSI) Caused by Gas Flow Force

$$\delta_{ST} = \frac{1.576 \ x \ 10^{-}4 \ (L - .5L_1) \ L_1 \ D^4 \ \rho_a \ f_w^2}{(D^4 - d^4)} = 69.907 \ PSI$$

6) Stress at Sensor Support (PSI) Caused by Vortex Shedding

$$\delta_{DYN} = \delta_{ST} \pm F_M \ \delta_{ST} = 69.91 \pm 48.5 \ PSI$$

7) Stress at Sensor Support Caused by Weight of Support

$$\delta_g = \frac{1.13\ L^2\ D}{D^2 + d^2} = 245.6\ PSI$$

8) Maximum Stress at Sensor Support

$$\delta_{MAX} = \delta_g + \delta_{DYN} = 315.5 \pm 48.5 = \ll 30{,}000\ ALLOWABLE\ PSI$$

No Failure at Maximum Flow Conditions

Example B: High Flare Gas Application Velocity

Process Pipe has 15.25" Diameter. $A = 1.2684\ FT^2$

D = 1" Series 450-16 Mass Flow Element

d = 0.87"

L = 15.35" + 2.234" + .78" = 18.36"

L_1 = 3.014" (Centerline of sensor placed at equal area location of pipe)

m = 80.975 lb*m*/Sec

P_a = 6 PSIG = 20.7 PSIA

P_s = 14.7 PSIA

MW = 53.85

T_a = 165 °F = 625 °R

T_s = 537 °R

δ = .01

Results

1) Natural Frequency

$$f_N = \frac{22{,}418\ \sqrt{D^2 + d^2}}{L^2} = 88.15\ Hz$$

2) Gas Density

$$\rho_a = .0748 \left(\frac{MW}{29}\right) x \left(\frac{P_a}{P_S}\right)\left(\frac{T_S}{T_a}\right) = .16804 \ lbm/Ft^3$$

3) Vortex Shedding Frequency

$$f_w = \frac{2.64 \ m}{\rho_a \ A \ D} = 1003 \ Hz$$

4) Force Magnification Factor

$$F_M = \frac{1}{\sqrt{\left(1 - \left(\frac{f_W}{f_N}\right)^2\right)^2 + \delta^2}} = .007784$$

5) Maximum Stress at Sensor Support (PSI) Caused by Gas Flow Force

$$\delta_{ST} = \frac{1.576 \ x \ 10^{-}4 \ (L - .5L_1) \ L_1 \ D^4 \ \rho_a \ f_W^2}{(D^4 - d^4)} = 3169 \ PSI$$

6) Stress at Sensor Support (PSI) Caused by Vortex Shedding

$$\delta_{DYN} = \delta_{ST} \pm F_M \ \delta_{ST} = 3169 \pm 24.67 \ PSI$$

7) Stress at Sensor Support Caused by Weight of Support

$$\delta_g = \frac{1.13 \ L^2 \ D}{D^2 + d^2} = 216.81 \ PSI$$

8) Maximum Stress at Sensor Support

$$\delta_{MAX} = \delta_g + \delta_{DYN} = 3386 \pm 24.67 \ PSI \ \text{«} \ 30{,}000 \ ALLOWABLE \ PSI$$

No Failure at Maximum Flow Conditions